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Relativistic Jets Emanating from a Black Hole Due to the Acceleration by the Magnetic Field Generated by the Accretion Disk

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When matter falls towards a black hole, it will form a disk around the black hole known as the accretion disk. Accretion disks have been observed to give rise to relativistic jets coming from the central object. The hypothesis is that the magnetic field produced by a charged accretion disk directs the outflow along the rotation axis of the central object, so that, when conditions are suitable, a jet will emerge from each face of the accretion disk. We have constructed a model of the magnetic field produced by an accretion disk around a black hole in an active galactic nucleus, and used it to show that particles coming off the accretion disk move in the z direction. A computer program was created to simulate the motion of a particle in the magnetic field. The particle is shown to leave its circular motion and move in the z direction.

Introduction

Einstein's theory of general relativity predicted the existence of black holes. A black hole is a spacetime region of strong gravitational field left after matter collapses. The strong gravitational field prevents anything from escaping, even light.[1] When matter falls towards a black hole, it will form a disk around the black hole known as the accretion disk.[2] Gravity causes some material from the accretion disk to fall into the black hole. Accretion disks have been observed to give rise to jets along their polar axes. Why these jets form is an unsolved problem in physics and one which is of interest to astrophysicists.

To explain the formation of relativistic jets along the axis of rotation of a black hole, we use two major computations. The first consists of computing the magnetic field (B-field) generated by the rotating and charged accretion disk around the black hole. For this computation, we use the Biot-Savart law. The accretion disk is approximated by a set of single rings**.** The program computes the B-field for each ring and sums over all the rings. Once the magnetic field was computed and verified, we then designed a second program that calculates the trajectory of a charged particle under the influence of the Lorentz force. Starting the particles in different positions along the accretion disk, the program allows us to calculate the trajectory upward and inward toward the z-axis. Both of these programs were written in Fortran language and graphed with Mathematica.

Magnetic Field

Single Loop

In order to compute the magnetic field generated by the spinning and charged accretion disk, we first considered the disk as the summation of many single rings. Such a ring is depicted in Figure 1, in the xy-plane, with radius R, centered on the origin. In this figure point P is an arbitrary location at which we wish to compute the magnetic field. In physics, the Biot-Savart law describes the magnetic field generated by an electric current.[3] The law provides us with an equation to calculate the B-field produced by each ring.

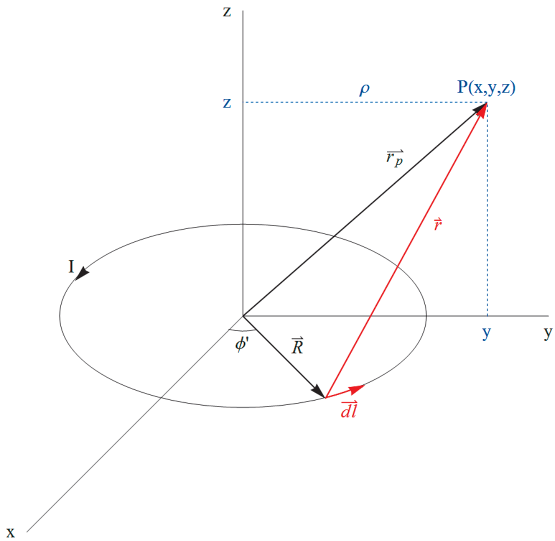

Figure 1
Current carrying ring of radius R in the xy-plane centered at the origin

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I\,\overrightarrow{dl}\times\vec{r}}{r^3} \tag{1}$$

where μ_0 is the vacuum permeability
I is the electric current in the loop
dl is the infinitesimal element of length of the source
R is the radius of the loop
$\vec{r}$ is the vector from source to detector
$\vec{r} = \vec{r_p} - \vec{R}$ where $\vec{r_p}$ is the detector position and $\vec{R}$ is the source position

For the specific case of a ring in the *xy*-plane, we have:

$$\overrightarrow{r_p} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{and} \quad \vec{R} = R \begin{pmatrix} \cos\phi' \\ \sin\phi' \\ 0 \end{pmatrix} d\phi' \qquad \text{(2) a,b,c}$$

$$r^2 = \left[(x - R\cos\phi')^2 + (y - R\sin\phi')^2 + z^2\right]^{1/2} \qquad (3)$$

$$\overrightarrow{dl} \times \vec{r} = R \begin{pmatrix} z\cos\phi' \\ z\sin\phi' \\ R - y\sin\phi' - x\cos\phi' \end{pmatrix} d\phi' \qquad (4)$$

Therefore

$$B_x = \frac{\mu_0 I R}{4\pi} \int_0^{2\pi} \frac{z\cos\phi'\, d\phi'}{\left[(x - R\cos\phi')^2 + (y - R\sin\phi')^2 + z^2\right]^{3/2}} \qquad (5)$$

$$B_y = \frac{\mu_0 I R}{4\pi} \int_0^{2\pi} \frac{z\sin\phi'\, d\phi'}{\left[(x - R\cos\phi')^2 + (y - R\sin\phi')^2 + z^2\right]^{3/2}} \qquad (6)$$

$$B_z = \frac{\mu_0 I R}{4\pi} \int_0^{2\pi} \frac{(R - y\sin\phi' - x\cos\phi')\, d\phi'}{\left[(x - R\cos\phi')^2 + (y - R\sin\phi')^2 + z^2\right]^{3/2}} \qquad (7)$$

The magnetic field in the x direction is determined to be zero as we have chosen field points, $\overrightarrow{r_p}$, in the *yz*-plane ($x=0$) and the loop is symmetrical about the z-axis. We then computed the magnitude of the magnetic field in the *yz*-plane in terms of y and z coordinates. Using our computer program, we computed these integrals given by Biot-Savart by summing the magnetic field over small portions of the loop (summation over the angle ϕ' with an increment of 1.0°). Figure 2 shows the lines of magnetic force for a single loop, interpolated from the magnetic field data given by our program. The values for the variables were chosen for graphical convenience.

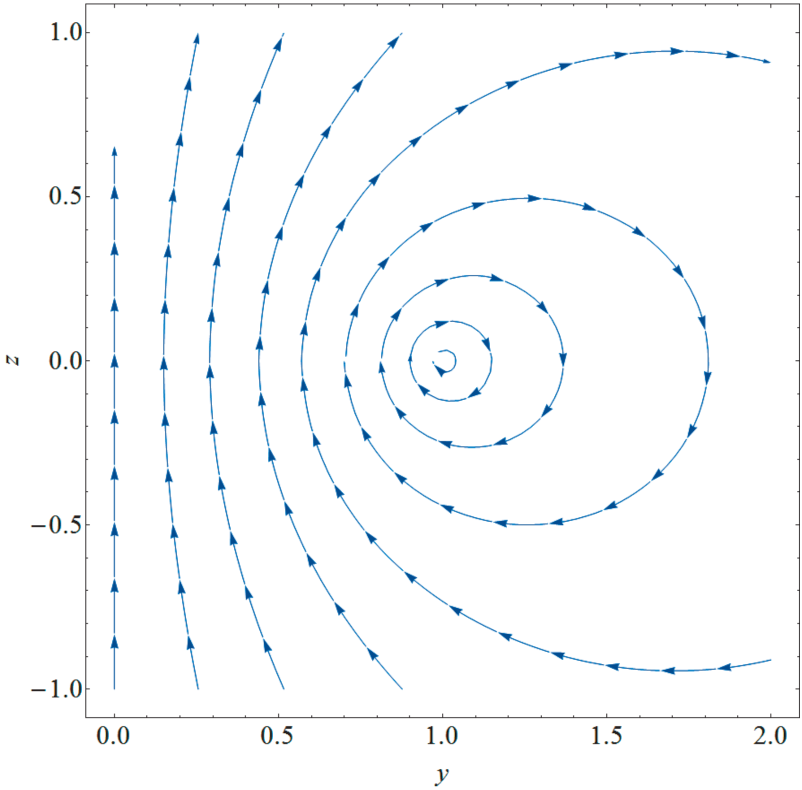

Figure 2

Lines of magnetic field in the *yz*-plane, for a current carrying loop of radius $R = 1.0$ and current $I = 1.0$

Accretion Disk

To obtain the magnetic field for a disk, we simply add the previous results for a certain amount of rings, each of different radius. In our case, we decided to have the disk span the distance from $r_{inner} = 1.0$ to $r_{outer} = 2.0$ and to use 100 rings (here the summation is over the radius with an increment of 1/100). Another characteristic specific to the accretion disk is that the current in each loop is inversely proportional to the square root of the radius because the speed of a particle in the accretion disk follows a Keplerian motion. The results are shown in Figure 3.

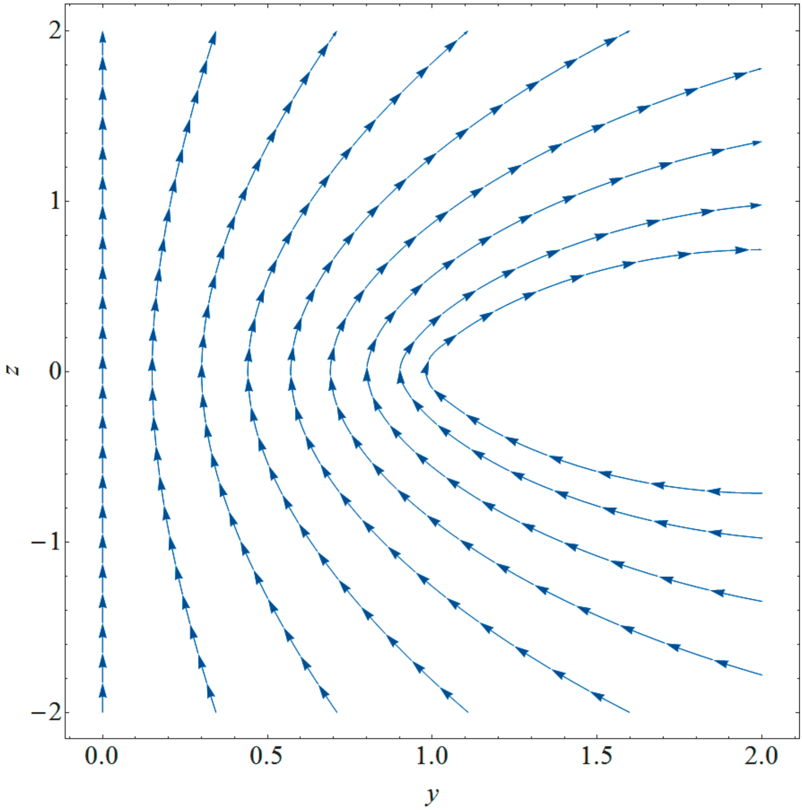

Figure 3
Lines of magnetic field for an accretion disk running from $r = 1.0$ to 2.0 and $I = 1/\sqrt{r}$.

The magnetic field lines are plotted in 2D, but can easily be thought of as a representation for the 3D image by rotating the Figure 3 around the z-axis.

Mathematical method to compute the Magnetic Field

In order to make sure that our program is correct, we mathematically computed the B-field for arbitrary points in space. Instead of reasoning on infinitesimal source of currents like in the Fortran program, we here deal with integral forms.[4] We also use cylindrical coordinates: P $(\rho,0,z)$. Due to the symmetry of the situation, we set $\phi = 0$. Before computing the magnetic field, we first compute the vector magnetic potential[3] $\vec{A}$:

$$\vec{A} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl}}{r} \tag{8}$$

where r is the distance between the point source and the detector. From the geometry of the situation (see Figure 1) we have:

$$r^2 = z^2 + \rho^2 + R^2 - 2 R \rho \cos\phi' \tag{9}$$

$$\vec{dl} = R \begin{pmatrix} -\sin\phi' \\ \cos\phi' \\ 0 \end{pmatrix} d\phi' \text{ (in Cartesian) ; } \qquad \vec{dl} = R\, d\phi' \, \vec{e_{\phi'}} \text{ (in cylindrical)} \tag{10}$$

where R is the radius of the ring, ϕ' is the angle from the axis to the source point, and ρ is the cylindrical radius to the point P in Figure 1. An expression for $\vec{A}$ in cylindrical coordinates (ρ, ϕ, z) is obtained, where ϕ is the angular coordinate of the detector position.

$$\vec{A} = \frac{\mu_0 I}{2\pi} \sqrt{\frac{R}{\rho}} \left[2 k^{-1} E_1(k) - 2 k^{-1} E_2(k) - k E_1(k) \right] \vec{e_\phi} \tag{11}$$

$E_1(k)$ and $E_2(k)$ are elliptic integrals of the first and the second kind, and are given by the following equations, where θ is an arbitrary integration variable:

$$k = \sqrt{\frac{4 R \rho}{z^2 + (R+\rho)^2}} \tag{12}$$

$$E_1(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2\theta}}, \qquad E_2(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2\theta}\; d\theta$$

(13) a, b

We note that $\vec{A}$ only lies on $\vec{e_\phi}$ which implies that $\vec{B}$ has a radial and z-components in cylindrical coordinates. $\vec{B}$ is obtained from the curl of $\vec{A}$:

$$\vec{B} = \vec{\nabla} \times \vec{A} = \frac{1}{\rho} \begin{vmatrix} \vec{e_\rho} & \vec{e_\phi} & \vec{e_z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} = \frac{1}{\rho} \left(\frac{\partial(\rho A_\phi)}{\partial z} \vec{e_\rho} + \frac{\partial(\rho A_\phi)}{\partial \rho} \vec{e_z} \right) = B_\rho \vec{e_\rho} + B_z \vec{e_z} \tag{14}$$

Equation (14) allows us to find an expression of B_ρ and B_z. We can use B_ρ to find expressions for B_x and B_y:

$$B_x = B_\rho \cos\phi \tag{15}$$

$$B_y = B_\rho \sin\phi \tag{16}$$

Therefore

$$B_x = \frac{\mu_0 z I}{2\pi \rho \sqrt{z^2 + (R+\rho)^2}} \left(\frac{R^2 + z^2 + \rho^2}{z^2 + (R-\rho)^2} E_2(k) - E_1(k) \right) \cos\phi \tag{17}$$

$$B_y = \frac{\mu_0 z I}{2\pi \rho \sqrt{z^2 + (R+\rho)^2}} \left(\frac{R^2 + z^2 + \rho^2}{z^2 + (R-\rho)^2} E_2(k) - E_1(k) \right) \sin\phi \tag{18}$$

$$B_z = \frac{\mu_0 I}{2\pi \sqrt{z^2 + (R+\rho)^2}} \left(\frac{R^2 - z^2 - \rho^2}{z^2 + (R-\rho)^2} E_2(k) - E_1(k) \right) \tag{19}$$

Since these equations characterize the *B*-field only for a single loop, our verification only deals with an accretion disk as the sum of two single rings. We then compute (B_x, B_y, B_z) for each of the two rings using equations 17, 18, and 19 and then take the sum to get the total *B*-field produced by the accretion disk. Finally, we compute the *B*-field using our Fortran program and compare the results with the curl of $\vec{A}$ outlined above. We checked four random points at different (x,y,z) and in the worst case, the *B*-field from the handwritten formula matches the Fortran program within five parts per million.

Particle Trajectory

The final model consists of a particle rotating just above the accretion disk and then, under the influence of the magnetic field, rising up and in towards the z-axis. The particle starts in the (y, z) plane at a small $z = 0.2$ and at $y = y_0$ with an initial velocity directed in the x-direction $\vec{v} = v_0\,\hat{i}$. We start the particle at a small $z = 0.2$ because the magnetic field diverges at $z = 0$.

In physics, the force on a particle due to electromagnetic fields is given by the Lorentz equation:

$$\vec{F} = \frac{d\vec{p}}{dt} = q\left(\vec{v}\times\vec{B}\right) \tag{20}$$

$$\begin{pmatrix} \Delta v_x/\Delta t \\ \Delta v_y/\Delta t \\ \Delta v_z/\Delta t \end{pmatrix} = \frac{q}{m}\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}\times\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \frac{q}{m}\begin{pmatrix} v_y B_z - v_z B_y \\ v_z B_x - v_x B_z \\ v_x B_y - v_y B_x \end{pmatrix} \tag{21}$$

If we consider Δt sufficiently small so that the velocity remains approximately constant over that amount of time, we can find the change in velocity:

$$\begin{aligned} \Delta v_x &= \tfrac{q}{m}\,(v_y B_z - v_z B_y)\,\Delta t \\ \Delta v_y &= \tfrac{q}{m}\,(v_z B_x - v_x B_z)\,\Delta t \\ \Delta v_z &= \tfrac{q}{m}\,(v_x B_y - v_y B_x)\,\Delta t \end{aligned} \tag{22}$$

For each step Δt, the program reads the position and the velocity of the previous step (the initial conditions). It then computes the magnetic field at this point in order to obtain, from which we get $\vec{v_f} = \vec{v_i} + \Delta\vec{v}$ and finally the new vector position: $\vec{R_f} = \vec{R_i} + \vec{v}\,\Delta t$. This operation is repeated over 100 steps, thus obtaining the trajectory of the particle. This program was run five times, each particle starting at a different positions on the y-axis. The conditions used:

$y_0 = 1.1;\ 1.2;\ 1.3;\ 1.4;\ 1.5$

$z_0 = 0.2$

$v_0 = 10^8\ \mathrm{m/s}$

$q/m = 1.76\times 10^{11}\ C/\mathrm{kg}$

$\Delta t = 5\times 10^{-10}\ s$

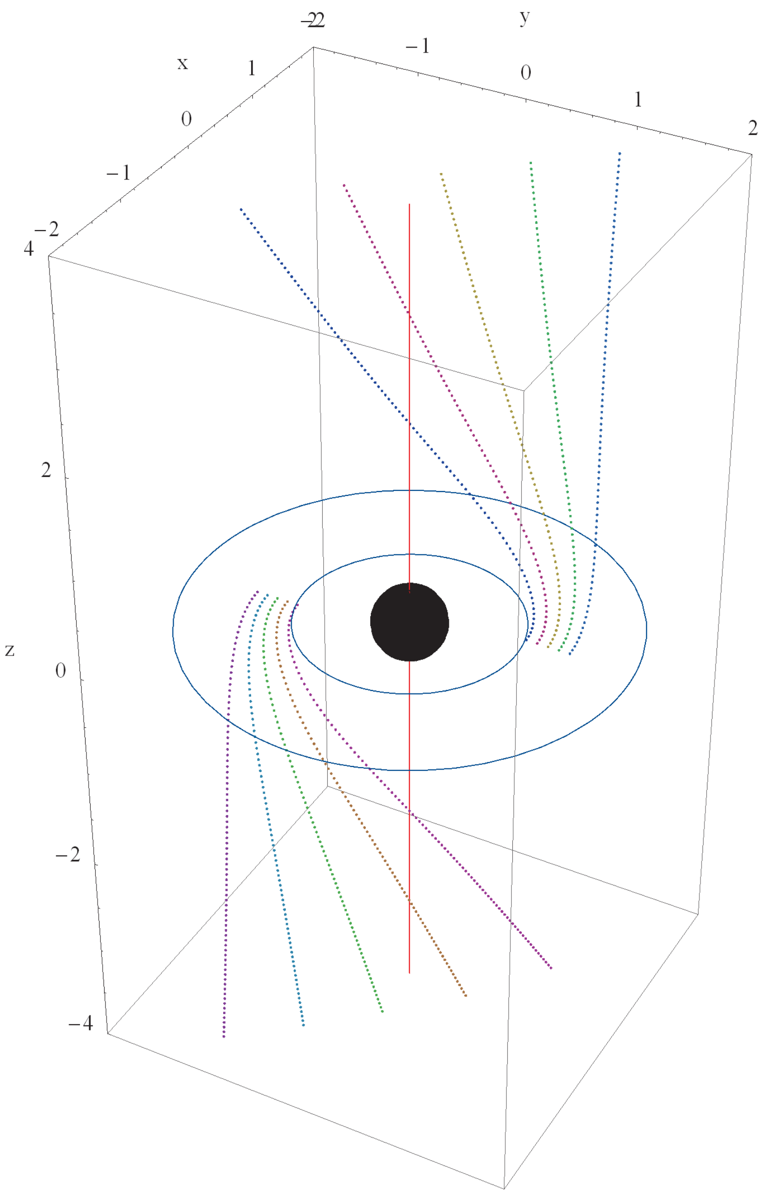

Figure 4
Three-dimensional image of the particle trajectory, due to the magnetic field of the accretion disk. In this case, the black hole, represented by its event horizon here (the limit of no return for any particle) is a Kerr Black Hole, which means it is spinning but it is not charged.

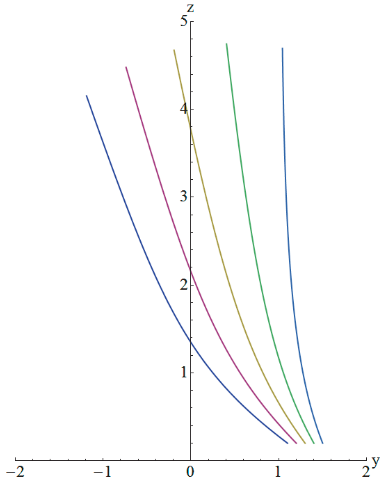

Figure 5
Particles' trajectory in the *yz*-plane

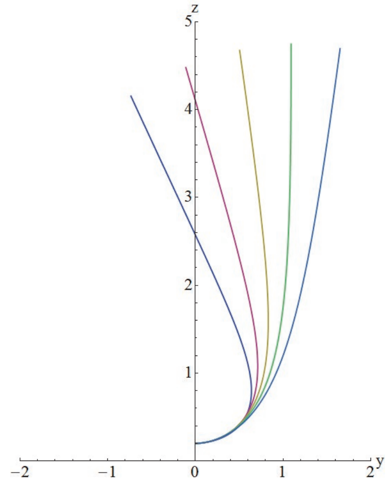

Figure 6
Particles' trajectory in the xz-plane

Figure 4 is the three-dimensional image of the particles trajectory. This figure illustrates the accretion disk with inner radius being the inside ring, $R=1$ and outer radius being the outside ring, $R=2$. Also illustrated is the black hole, centered at the origin and the rotation axis of the black hole, in the z-direction. Our three-dimensional image plotted in Mathematica shows how the particles move due to the magnetic field produced by the accretion disk. To better visualize the trajectory of the particles, Figures 5 and 6 show the two-dimensional motion in the yz-plane and xz-plane respectively.

Formation of relativistic jets

As we can see from Figures 4 through 6, the particles leave the accretion disk and move in an upward/downward direction, along the axis of rotation of the black hole. The particles tend to move on a straight line when they get far enough away from the accretion disk, some passing through the z-axis. So how do the particles stay near the z-axis? As we get close to the black hole, space and time are twisted due to the angular momentum of the black hole and of the accretion disk. According to the current model established by Roger Blandford[5], the magnetic field lines also are twisted around the z-axis, resulting in an accumulation of the particles near the axis of rotation of the black hole, which could explain why the relativistic jets are long and narrow. Therefore, we have constructed a model which demonstrates that particles can be accelerated vertically away from the accretion disk and inward toward the z-axis. Together with the frame-dragging effects of general relativity, this results in a relativistic jet.

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